Foundations 110 – Ms. Follett

Review of Factoring Quadratic Trinomials ax^2 + bx + c

ax^2 + bx + c
a: coefficient of x^2 b: coefficient of x c: constant term

Recall that expanding and factoring are opposite procedures:

Expanding	**Factoring**
$(2x+9)(2x+1)$	$4x^2+20x+9$
$2x(2x+1)+9(2x+1)$	$4x^2+2x+18x+9$
$4x^2+2x+18x+9$	$\dfrac{4x^2+2x}{2x}+\dfrac{18x+9}{9}$
$4x^2+20x+9$	$2x(2x+1)+9(2x+1)$
	$(2x+9)(2x+1)$

Types of factoring learned in NRF 10:

1. **Common Factoring (GCF – Greatest Common Factor)**

   Example: Factor 5x^2 – 10x + 15

   There is a GCF of 5, therefore we take 5 out of the expression to factor it (divide each term by 5) and determine what factors remain.

   $5x^2 - 10x + 15 = 5(? - ? + ?)$
   $= 5(x^2 - 2x + 3)$

2. **Factoring Quadratic Trinomials with a = 1**

   Example: Factor x^2 – 11x + 24

   Since a = 1, we are looking for two numbers that multiply to give c = +24 and the same two numbers must add to give b = -11. What are the two numbers? -8 and -3.

   $x^2 - 11x + 24 = (x \quad ?)(x \quad ?)$
   $= (x - 8)(x - 3)$

3. Difference of Squares

Example: Factor $x^2 - 16$

You will notice that both terms are perfect squares, there is no middle x term, and the operation is subtraction (*difference* of squares). In order to cancel the middle term ($0x$) we will need opposite terms. Therefore, to factor a difference of squares you take the square root of each term of the expression as the first and last terms in your binomial factors and one factor will be addition and the other will be subtraction.

$$\begin{aligned} x^2 - 16 &= (? \quad ?)(? \quad ?) \\ &= (x \quad 4)(x \quad 4) \\ &= (x + 4)(x - 4) \end{aligned}$$

4. Combination Factoring

Always check for a GCF before trying any other type of factoring!

Example 1: Factor $75x^2 - 147$

There is a GCF of 3 so we will first take out the GCF.

$$75x^2 - 147 = 3(25x^2 - 49)$$

Then we notice we are left with a *difference of squares*, so we factor further.

$$\begin{aligned} 75x^2 - 147 &= 3(25x^2 - 49) \\ &= 3(5x + 7)(5x - 7) \end{aligned}$$

Example 2: Factor $4x^2 - 8x - 140$

There is a GCF of 4 so we will first take out the GCF.

$$4x^2 - 8x - 140 = 4(x^2 - 2x - 35)$$

Then we notice we are left with a *quadratic trinomial with $a = 1$*, so if we can factor further, we are looking for two numbers that multiply to give c = -35 and the same two numbers have to add to give b = -2. Are there such numbers? Yes, -7 and +5. Therefore, we factor further.

$$\begin{aligned} 4x^2 - 8x - 140 &= 4(x^2 - 2x - 35) \\ &= 4(x - 7)(x + 5) \end{aligned}$$

5. Factoring Quadratic Trinomials with a ≠ 1 by Decomposition

Example: Factor $8x^2 - 10x - 3$

Since a ≠ 1, the process to factor this quadratic trinomial involves more steps than when a = 1.

Step 1: Multiply a and c together. (8)(-3) = -24

Step 2: Find two numbers that multiply to give (a)(c) = -24 and the same two numbers must add to give b = -10. What are the two numbers? +2 and -12.

Step 3: *Decompose* the middle x term of the original trinomial into two x terms using those two numbers.

$8x^2 - 10x - 3 = 8x^2 + 2x - 12x - 3$ (the order of the middle x terms does not matter)

Step 4: Common factoring – Remove the GCF out of the *first* two terms and the GCF out of the *last* two terms.

$$\begin{aligned} 8x^2 - 10x - 3 &= 8x^2 + 2x - 12x - 3 \\ &= 2x(4x + 1) - 3(4x + 1) \end{aligned}$$

(notice your remaining factors in the brackets are the same, as they *must* be)

Step 5: Gather the GCF's together as one binomial factor, and the second binomial factor is the common one in the brackets.

$$\begin{aligned} 8x^2 - 10x - 3 &= 8x^2 + 2x - 12x - 3 \\ &= 2x(4x + 1) - 3(4x + 1) \\ &= (2x - 3)(4x + 1) \end{aligned}$$

6. Combination Factoring

Again, always check for a GCF before trying any other type of factoring!

Example: Factor $6x^2 - 21x + 9$

There is a GCF of 3 so we will first take out the GCF.

$$6x^2 - 21x + 9 = 3(2x^2 - 7x + 3)$$

Then we notice we are left with a *quadratic trinomial with a ≠ 1*, so if we can factor further, we are looking for two numbers that multiply to give (a)(c) = +6 and the same two numbers have to add to give b = -7. Are there such numbers? Yes, -6 and -1. Therefore, we factor further (*by decomposition*).

$6x^2 - 21x + 9 = 3(2x^2 - 7x + 3)$	(remove GCF)
$= 3(2x^2 - 6x - 1x + 3)$	(decompose middle term)
$= 3[2x(x - 3) - 1(x - 3)]$	(remove GCF from first two and last two terms)
$= 3(2x - 1)(x - 3)$	(gather GCF terms as one factor and the second factor is the common one in brackets)