Solving Quadratic Functions:

- table of values
- partial factoring

graph the quadratic and find other information about it

- factoring
- quadratic formula

find the roots/x-intercepts

$ax^2 + bx + c = 0$

Solving Problems Using Quadratic Models

Example 1:

A Frisbee is thrown into the air and its height in feet as a function of time in seconds is given by the function $h(t) = -2t^2 + 3t + 5$.

a) How high above the ground is the Frisbee just before it is thrown?

let $t = 0$

$h(0) = -2(0)^2 + 3(0) + 5$
$= 5$

The Frisbee is 5ft high before being thrown.

b) How high is the Frisbee after 2 seconds?

let $t = 2$

$h(2) = -2(2)^2 + 3(2) + 5$
$= 3$

After 2 seconds, the frisbee is 3 ft high.

c) When does the Frisbee reach a height of 4 ft?

let $h(t) = 4$

$4 = -2t^2 + 3t + 5$

$2t^2 - 3t - 1 = 0$

-2, -3

cannot be factored

The frisbee reaches 4 ft at 1.78 seconds.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-1)}}{2(2)}$

$x = \frac{3 \pm \sqrt{17}}{4}$

$x = \frac{3 + \sqrt{17}}{4}$ $\quad$ $x = \frac{3 - \sqrt{17}}{4}$

$x \doteq 1.78$ $\quad$ $x \doteq -0.28$ Inadmissable

d) When does the Frisbee hit the ground? roots/ x-int's

let $h(t) = 0$

-10, -2, 5

$0 = -2t^2 + 3t + 5$

$0 = -2t^2 - 2t + 5t + 5$

$0 = -2t(t+1) + 5(t+1)$

$0 = (-2t + 5)(t+1)$

$-2t + 5 = 0$
$-2t = -5$
$t = \frac{-5}{-2}$
$t = 2.5$

$t + 1 = 0$
$t = -1$
Inadmissable

The frisbee hits the ground after 2.5 seconds.

e) How long does it take for the Frisbee to reach its maximum height?

* axis of symmetry *

$\frac{2.5 + (-1)}{2} = 0.75$

The frisbee reaches its max height at $\frac{3}{4}$'s second.

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f) What maximum height is reached by the Frisbee? * vertex (0.75, ?)

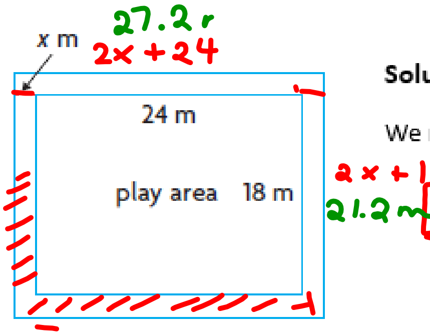
$h(0.75) = -2(0.75)^2 + 3(0.75) + 5$
$= -1.125 + 2.25 + 5$
$= 6.125$

The frisbee reaches a max height of 6.125 ft.

Example 3:

Ian has been hired to lay a path of uniform width, using crushed rock, around a 24 m by 18 m rectangular play area. He has enough crushed rock to cover 145 m^2. If Ian uses all the crushed rock, how wide will the path be? Verify your solution.

$576.64\text{ m}^2 - 432\text{ m}^2 = 144.64\text{ m}^2$



Solution:

We need to determine the width of the path that will result in an area of 145 m^2.

Area of path = Total area – Play area
$A(x)$

Area of path =

Substitute 145 m^2 for the area of the path and solve the equation for x:

$$A(x) = (2x+24)(2x+18) - (24)(18)$$
$$A(x) = 4x^2 + 36x + 48x + 432 - 432$$
$$A(x) = 4x^2 + 84x$$

Let $A(x) = 145$

$$145 = 4x^2 + 84x$$
$$0 = 4x^2 + 84x - 145$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-84 \pm \sqrt{84^2 - 4(4)(-145)}}{2(4)}$$
$$x = \frac{-84 \pm \sqrt{9376}}{8}$$

$$x = \frac{-84 + \sqrt{9376}}{8}$$
$$\boxed{\doteq 1.6}$$

$$x = \frac{-84 - \sqrt{9376}}{8}$$
$$\doteq -22.6$$
Inadmissable

The width of the path is 1.6 m.

Example 4:

A store rents an average of 750 video games each month at the current price of \$4.50 per game. The owners of the store want to raise the rental price, however, for every \$1 price increase, they know that they will rent 30 fewer games each month.

a) Let p equal the number of \$1 price increases. Write the function that relates p to the revenue, r.

$R = (4.50 + p)(750 - 30p)$

$\$4.50 + p \rightarrow$ current price + price increase

$750 - 30p \rightarrow$ current number of rentals − 30 fewer per price increase

b) Is it possible for the owners to increase the rental price enough to generate a revenue of \$7000 per month?

Let $R = \$7000$

$7000 = (4.50 + p)(750 - 30p)$

$7000 = 3375 - 135p + 750p - 30p^2$

$7000 = \boxed{3375 + 615p - 30p^2}$

$30p^2 - 615p - 3375 + 7000 = 0$

$\frac{30p^2 - 615p + 3625 = 0}{5}$

$6p^2 - 123p + 725 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-123) \pm \sqrt{(-123)^2 - 4(6)(725)}}{2(6)}$

$= \frac{123 \pm \sqrt{-2271}}{12}$

$\sqrt{-2271}$ is not a real number

$\therefore$ no real solutions and so it is not possible for the store to generate a revenue of \$7000 per month by increasing the rental rate.

c) What should the owners charge per game in order to maximize revenue?

axis of symmetry

vertex

$\frac{0 = -30p^2 + 615p + 3375}{-15}$

$0 = 2p^2 - 41p - 225$

$0 = p(2p - 41) - 225$

$0 = p \qquad 0 = 2p - 41$

$41 = 2p$

$20.5 = p$

axis of sym $x = 10.25$

$\frac{0 + 20.5}{2} = 10.25$

Charge: $\$4.50 + p$

$= \$4.50 + 10.25$

$= \$14.75$

Max Revenue

$R = -30(10.25)^2 + 615(10.25) + 3375$

$= \$6526.875$

Vertex $(10.25, 6526.875)$

The owners should charge \$14.75 per game to get a max revenue of \$6526.88.

(However if they charge \$14.75 per game they will probably have few rentals.)