Foundations 11 Exam Review (January 2014) ******SOLUTIONS*****

Chapter 1 - Logical Reasoning

1. Determine the number that should be in the centre of Figure4.

1		2	4		6	3		5	2		1
	24			192			450			**?**	
4		3	4		2	6		5	4		3
	Figure 1			Figure 2			Figure 3			Figure 4	

2. a) Write a reasonable conjecture about the sum of three odd integers.
   b) Use deductive reasoning to prove that the sum of two even numbers and one odd number will be an odd number.

3. Find a counterexample for each of the following conjectures.
   a) When you add a multiple of 6 and a multiple of 9, the sum will be a multiple of 6.
   b) The square of a number is always greater than the number.

4. The three little pigs built three houses: one of straw, one of sticks, and one of bricks. By reading the six clues, deduce which pig built each house, the size of each house, and the town in which each house was located.

Clues

- Penny Pig did not build a brick house.
- The straw house was not medium in size.
- Peter Pig's house was made of sticks, and it was neither medium nor small in size.
- Patricia Pig built her house in Pleasantville.
- The house in Hillsdale was large.
- One house was in a town called Riverview.

Solutions: Chapter 1 – Logical Reasoning

1. 24

2. a) Sum will be odd b) Deductive Proof

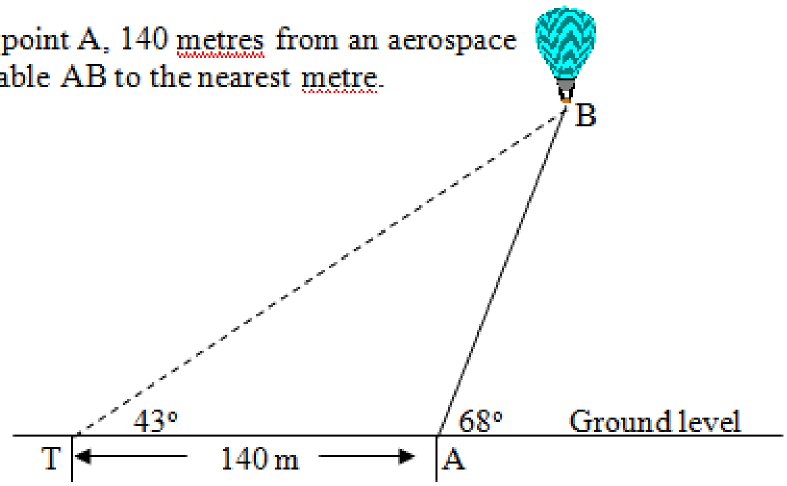
3. a) 15 (Sum of 9 and 6 is 15 BUT 15 is NOT divisible by 6)
b) 1 (1^1 = 1 which is not greater than 1)

4. Penny Pig → Small House (Straw) → Riverview
Patricia Pig → Medium House (Brick) → Pleasantville
Peter Pig → Large House (Sticks) → Hillsdale

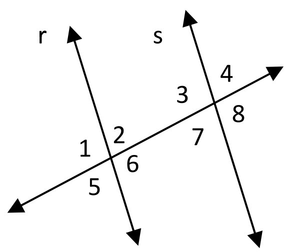
Chapters 2, 3 and 4 – Geometry and Trigonometry

1. A weather balloon is anchored to the ground at point A, 140 metres from an aerospace tracking station at point T. Find the length of the cable AB to the nearest metre. (value 3)

1. AB = 226m



2. Given that line r is parallel to line s. State the angles that may be referred to as:

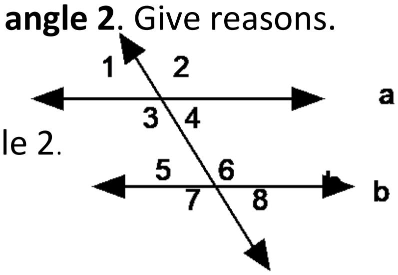


a) corresponding angles
b) alternate interior angles
c) alternate exterior angles
d) co-interior angles
e) state the relationship between the angles in a) & d)

2. Many Answers

3. a) If angle 4 above is 112° determine the measure of **angle 1 and angle 2**. Give reasons.

b) In the diagram at right a || b. Prove that angle 3 is equal to angle 2.

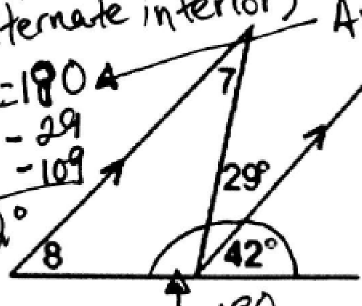


3. a) <1 = 112°, <2 = 68° b) Many Answers

4. Find the value of the angle(s) indicated in each of the following diagrams. Be sure to provide valid reasons for each answer.

a)

7 = 29°
(Alternate interior)
∠8 = 180
− 29
−109
42°

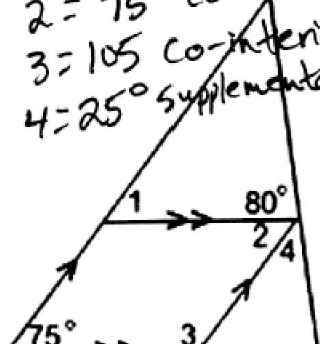


180
− 29
− 42
109

b)

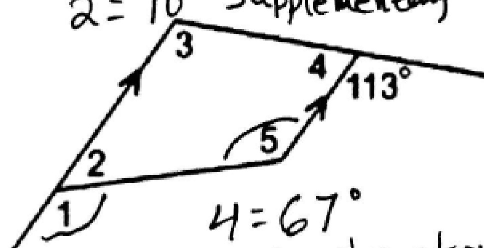
Angle sum of a Δ.

1 = 75° corresponding
2 = 75° co-interior
3 = 105 co-interior
4 = 25° supplementary



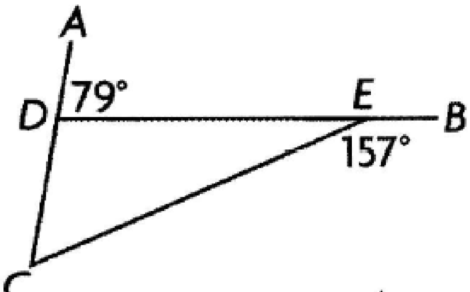
c)

3 = 113° : Corresponding
1 = 5 = 110° Alternate interior
2 = 70° Supplementary



4 = 67° supplementary
3 = 113°

d) Find the measure of all angles in ΔCDE.

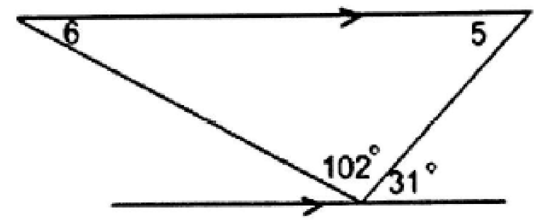


$\angle CDE = 101$ Supplementary

$\angle DEC = 23$ supplementary

$\angle DCE = 56$ Angle sum of Δ

e) Find the measures of angles 5 & 6

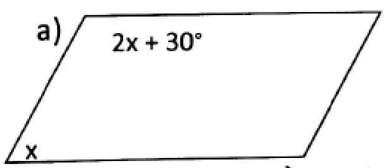


$\angle 5 = 31°$ Alternate interior

$\angle 6 = 47°$ Angle sum of Δ

5. Solve for x in each of the following. Provide clear reasons for your answers.

a)



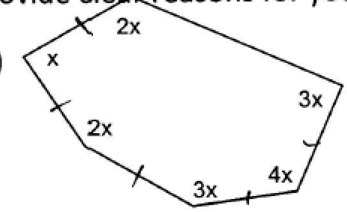
$S(n) = (2x+30) + (2x+30) + x + x$

$360° = 6x + 60$

$300 = 6x$

$\boxed{x = 50°}$

b)



$S(n) = (n-2)\,180$ n = # of sides

$S(n) = 180(6-2)$

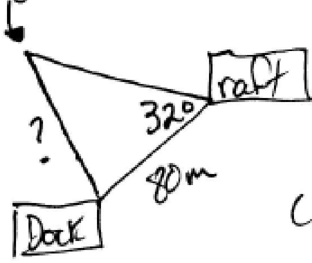
$S(n) = 720$

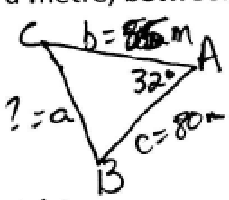
$S(n) = x + 2x + 2x + 3x + 4x + 3x$

$\frac{720}{15} = \frac{15x}{15}$

$\boxed{x = 48°}$

6. A swimmer leaves the dock and swims toward a raft 80 m away. After reaching the raft, she changes direction and swims another 55 m. She then stops and treads water. As measured from the raft, the angle between the line of sight to the dock and the line of sight to the swimmer's current position is 32°. Draw a diagram and determine the *shortest* possible distance, to the nearest tenth of a metre, between the dock and where the swimmer is treading water.





SAS

Cosine Law

$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 55^2 + 80^2 - 2(55)(80)\cos 32°$

$a^2 = 9425 - 7462.82$

$a^2 = 1962.18$

$a = \sqrt{1962.18}$

$\boxed{a = 44.3\text{m}}$

7. $x = 180° - 75° - 47° = 58°$

$a = 180° - 75° = 105°$

$$\frac{c}{\sin 58°} = \frac{15.5}{\sin 75°}$$

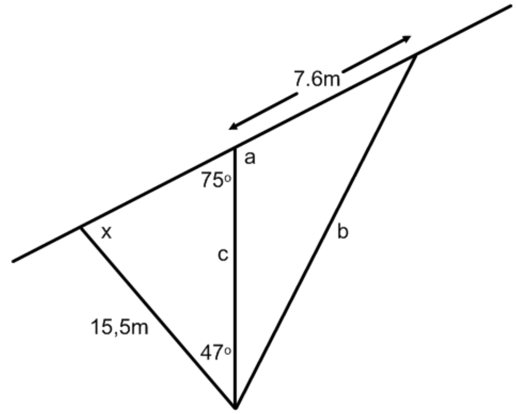
$$c = \frac{15.5\sin 58°}{\sin 75°} \doteq 13.61\text{m}$$

$$\text{b}^2 = a^2 + c^2 - 2ac\cos B$$

$$= (7.6)^2 + (13.61)^2 - 2(7.6)(13.61)\cos 105°$$

$$= 296.5345...$$

$$b \doteq 17.2\text{m}$$



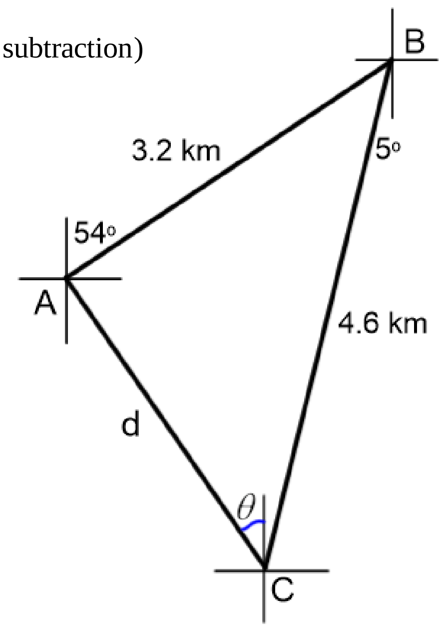
8 a.

$\angle ABC = 54^o - 5^o = 49^o$ (alternate interior angles & angle subtraction)

$$b^2 = 3.2^2 + 4.6^2 - 2(3.2)(4.6)\cos 49^o$$

$$= 12.0856...$$

$$b = 3.5\ km$$



b.

$$\angle C = \angle ACB$$

$$\frac{\sin C}{3.2} = \frac{\sin 49^o}{3.5}$$

$$\sin C = \frac{\sin 49^o}{3.5} \times 3.2 = 0.6900$$

$$C \doteq 44^o$$

$$\theta = 44^o - 5^o = 39^o$$

$$bearing:\ N39^oW$$

Chapter 5 – Linear Inequalities

Things to note:

Dashed lines will represent < or > signs

Solid lines will represent ≤ or ≥ signs

All points in the shaded region, if plugged into the equation will be TRUE

All points in the non-shaded region, if plugged into the equation will be FALSE

1. Given the inequality $2x + 4y \geq 16$. a) Dashed line or solid line? b) Rewrite the inequality into slope-intercept form. (y = mx + b) c) Graph the boundary line. d) Shade above or below the line? (Test point). **Try to use (0,0) where possible.**

$$-2x \quad 2x + \boxed{4y} \geq 16 \quad -2x$$

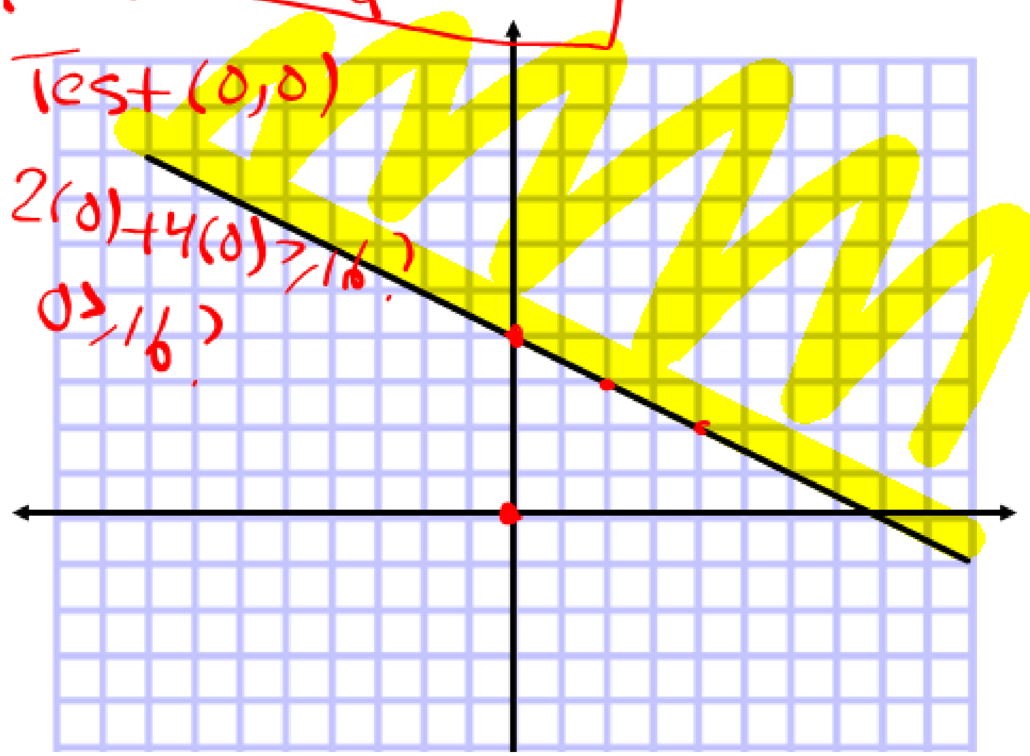
$$\frac{4y}{4} \geq \frac{-2x}{4} + \frac{16}{4}$$

$$\boxed{y \geq -\frac{1}{2}x + 4}$$

Test (0,0)

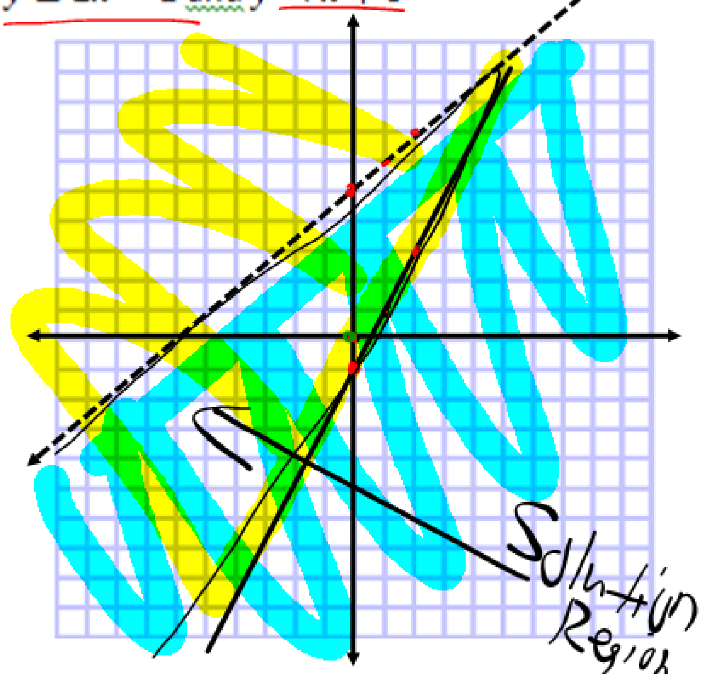
$2(0) + 4(0) \geq 16$?

$0 \geq 16$?



2. Determine the solution set for the following system of inequalities:

$y \geq 2x - 1$ and $y < x + 5$



Test pt $(0,0)$

$0 \geq 2(0) - 1$

$0 \geq -1$? yes

Test pt $(0,0)$

$0 < 0 + 5$

$0 < 5$? yes

Set of all possible solutions!

3. Determine the solution set for: $\{(x, y) \mid y \geq -\frac{3}{2}x + 3, x \in R, y \in R\}$ and $\{(x, y) \mid y \geq 6x + 1, x \in R, y \in R\}$

→ $m = -\frac{3}{2}$, y-int: 3

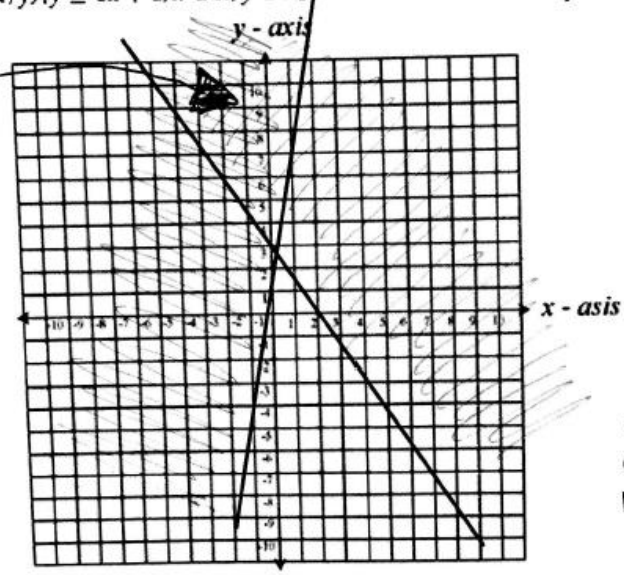
Test pt $(0,0)$

$0 \geq -\frac{3}{2}(0) + 3$

$0 \geq 3$? No

Shade above boundry line (Including the line)

Solution region



$y \geq 6x + 1$

$m = \frac{6}{1}$ y-int: 1

Test pt $(0,0)$

$0 \geq 6(0) + 1$

$0 \geq 1$? NO

(Include the line)

4. On a flight between Calgary and Thunder Bay, there are business and economy seats. At capacity, the airplane can hold no more than 133 passengers. Also, no fewer that 124 economy seats are sold and no more that 5 business class seats are sold. The airliner charges \$624 for each business class seat and \$239 for each economy seat. Let B represent the number of business class seats sold. Let E represent the number of economy seats sold. Provide 4 constraints for this problem as inequalties.

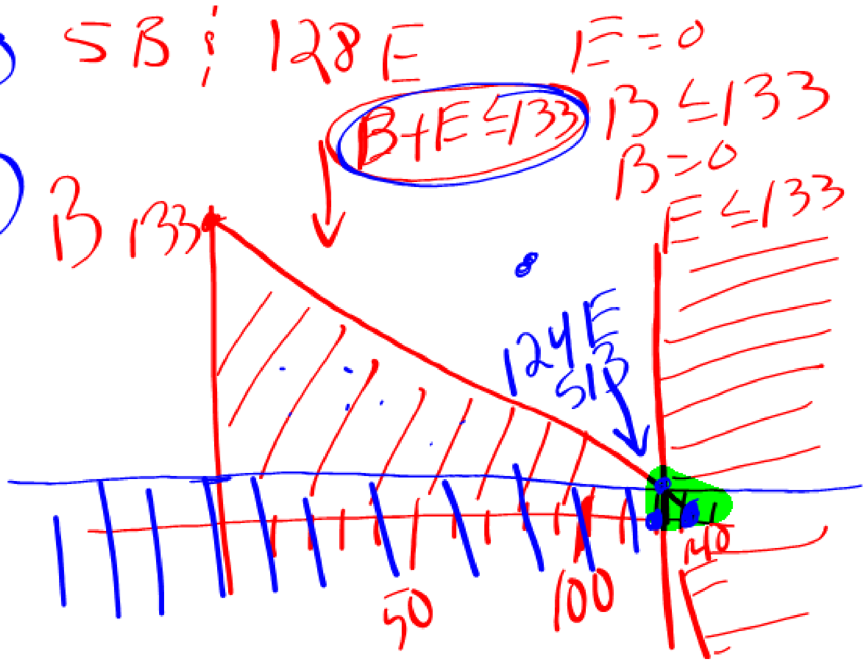
124 Economny seats & 5 Business class seats will yeild the greates revenue.

\$624 (5) + \$239 (124) = \$32756

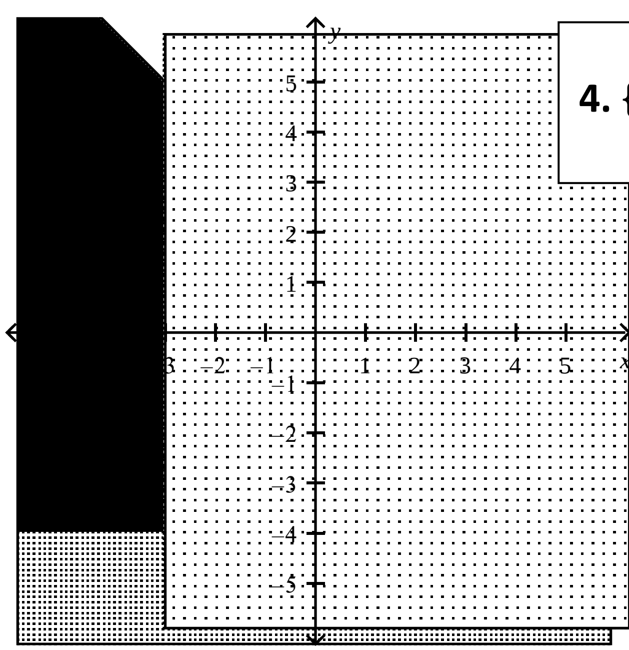
① $B + E \leq 133$

② $E \geq 124$

③ $B \leq 5$



5. What system of linear inequalities is shown here? Write your answer in set notation.



4. $\{(x, y) \mid x + y \leq 2, x > -3, x \in \mathbf{R}, y \in \mathbf{R}\}$

Chapter 6 – Quadratic Functions and Equations

1. Does the parabola $y = -2x^2 + 6x - 5$ open up or down? How do you know?

Down. Negative coefficient of "a"

2. For each function below: i) State the direction of opening ii) Determine the x and y-intercepts **(factor)** iii) Axis of Symmetry iv) Find the vertex v) Graph

a) $y = x^2 - 3x - 4$

b) $y = x^2 - 10x + 16$

a) i) up

ii) y-int $= -4$

x-int: 4 & -1

$y = x^2 - 3x - 4$

$y = (x-4)(x+1)$

$0 = (x-4)(x+1)$

$x - 4 = 0$ $\quad$ $x + 1 = 0$

$\boxed{x=4}$ $\quad$ $\boxed{x=-1}$

iii) AOS:

$x = \frac{4 + -1}{2}$

$x = 1.5 \Rightarrow$ Vertex: $y = (1.5)^2 - 3(1.5) - 4$

$y = -6.25$

$(1.5, -6.25)$

b)

i) up

ii) y-int: 16

$y = (x-8)(x-2)$

$0 = (x-8)(x-2)$

$x = 8$ $\quad$ $x = 2$

iii) AOS:

$x = \frac{8+2}{2}$

$x = 5 \Rightarrow$ Vertex

$y = (5)^2 - 10(5) + 16$

$y = -9$

$(5, -9)$

3. For the function $y = -x^2 + 6x - 3$:

3. For the function $y = -x^2 + 6x - 3$:

(a, b, c)

- Use partial factoring to find two points on the parabola
- Find the vertex
- Draw the graph of the quadratic function
- Use the Discriminant to determine the number of roots

Discriminant:

$b^2 - 4ac$

$6^2 - 4(-1)(-3)$

$36 - (12) = 24 > 0$

(2 roots

$y = (-x^2 + 6x) - 3$

$y = -x(x-6) - 3$

$0 = -x$ $\quad$ $0 = x - 6$

$\boxed{x=0}$ $\quad$ $\boxed{x=6}$

$f(0) = -3$

$f(6) = -(6)^2 + 6(6) - 3$

$f(6) = -3$

2 pts:

$(0, -3)$ & $(6, -3)$

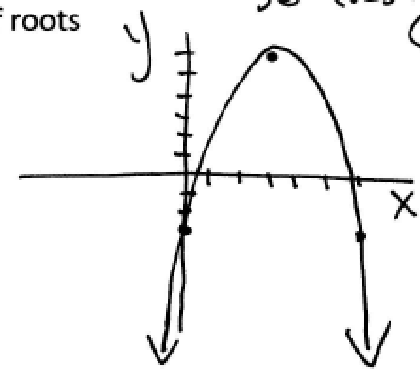
AOS: $x = \frac{0+6}{2}$

$\boxed{x=3}$

Vertex: $\rightarrow (3, 6)$

$f(3) = -(3)^2 + 6(3) - 3$

$f(3) = 6$



4. A flare is often used as a signal to attract rescue personnel in an emergency. When a flare is shot into the air, its height, $h(t)$, in meters, over time, t, in seconds can be modeled by $h(t) = -5t^2 + 120t$

a) Identify the x and y intercepts of the parabola
b) When did the flare reach its maximum height? What was the maximum height?
c) What was the height of the flare after 15s?
d) State the domain and range of the function.

4. a) y-int: 0, x-intercepts: (0,0) and (24,0) b) 12 seconds, 720 metres c) 675 metres **d) Domain: {x ϵ R, 0 ≤ x ≤ 24} / Range: {y ϵ R, 0 ≤ y ≤ 720}**

5. Use the quadratic formula to solve: $2x^2 - 5x - 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2-4(2)(-3)}}{2(2)}$$

$$x = \frac{5 \pm \sqrt{25+24}}{4}$$

$$x = \frac{5 \pm \sqrt{49}}{4}$$

$$x = \frac{5 \pm 7}{4}$$

$$x = \frac{12}{4} \ \& \ \frac{-2}{4}$$

$$\boxed{x = 3 \ \& \ -\frac{1}{2}}$$

6. Show how partial factoring can be used to find the vertex of the function:

f(x)= x^2- 8x + 13

$$f(x) = (x^2-8x) + 13$$

$$0 = x(x-8)$$

$$x=0 \qquad x=8$$

$$\boxed{f(0) = 13}$$

$$f(8) = (8)^2-8(8)+13$$

$$\boxed{f(8)=13}$$

$$(0,13) \ \& \ (8,13)$$

Same y-value: (0,13) & (8,13)

$$\text{AOS}: x = \frac{0+8}{2}$$

$$\boxed{x=4}$$

$$f(4) = (4)^2-8(4)+13$$
$$= 16-32+13$$
$$f(4) = -3$$

Vertex: (4, -3)

7. A cannon fires a ball which travels in a trajectory modeled by the function

$h(x) = -0.5x^2 + 15x + 2$ where $h(x)$ is the ball's height in meters, and x is the horizontal distance travelled in meters. a) How high is the end of the barrel of the cannon? b) How long ~~does it~~ of a take for the cannon ball to hit the ground?

what is the horizontal distance of the ball when the ball hits the ground?

$h(x) = -0.5x^2 + 15x + 2$, $h(x)$ = height (m)
x = horizontal

a) The height of the cannon is when the horizontal distance = 0

$h(0) = 2$

∴ The height of the cannon (end of the barrel = 2m)

b) When $h(x) = 0$, the horizontal distance has 2 values

8. Graph the equation $y = 5x^2 - 10x - 15$ by finding the **x-intercepts**, **y-intercepts** and the **vertex**. Also, determine the domain and the range for the function.

Factored form:

$y = 5(x^2 - 2x - 3)$

$y = 5(x-3)(x+1)$

$0 = 5(x-3)(x+1)$

$(x-3) = 0 \quad (x+1) = 0$

$x = 3 \quad x = -1$

(x-int's)

y-int: -15

AOS: $x = \frac{3 + -1}{2}$

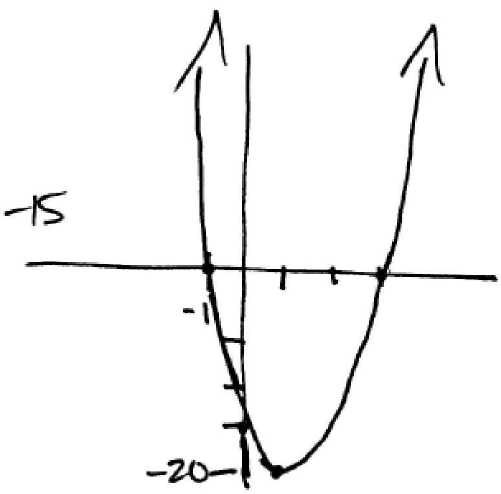
$x = 1$

$f(1) = 5(1)^2 - 10(1) - 15$

$f(1) = 5 - 10 - 15$

$f(1) = -20$

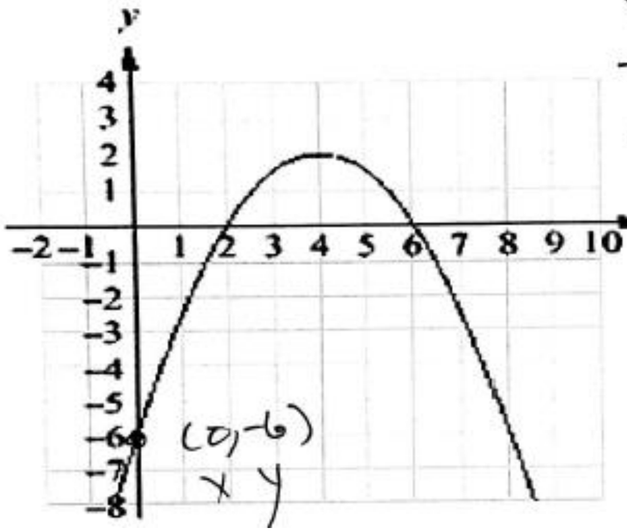
Vertex: $(1, -20)$



D: $\{x \mid x \in R\}$

R: $\{y \mid y \geq -20, y \in R\}$

9. Given the parabola below, determine the **factored form** and **standard form** that represents the quadratic.



$\hookrightarrow y=a(x-r)(x-s)$

$y=a(x-2)(x-6)$

$-6=a(0-2)(0-6)$

$-6=a(-2)(-6)$

$\frac{-6}{12}=\frac{12a}{12}$

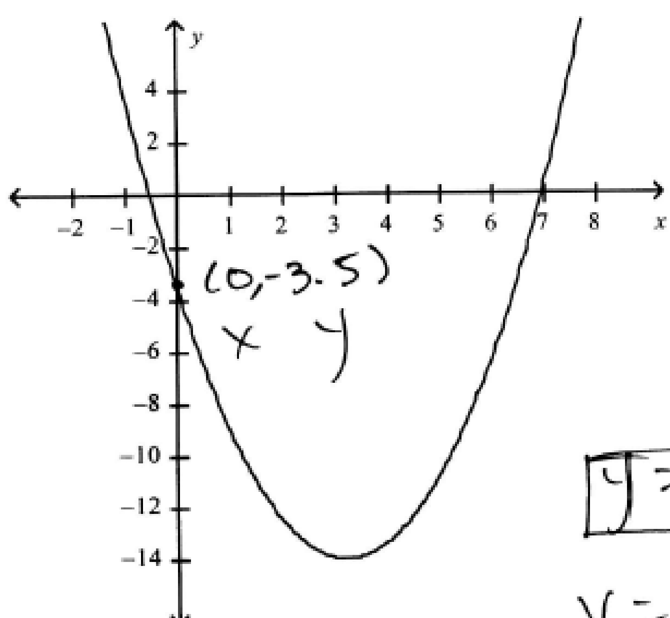
$a=-\frac{1}{2}$

$\boxed{y=-\frac{1}{2}(x-2)(x-6)}$

$y=-\frac{1}{2}(x^2-8x+12)$

$\boxed{y=-\frac{1}{2}x^2+4x-6}$

10. Given the parabola below, determine the **factored form** and **standard form** that represents the quadratic when the y-int is -3.5



$\hookrightarrow y=a(x-r)(x-s)$

$y=a(x+0.5)(x-7)$

$-3.5=a(0+0.5)(0-7)$

$-3.5=a(0.5)(-7)$

$\frac{-3.5}{-3.5}=\frac{a(-3.5)}{-3.5}$

$a=1$

$\boxed{y=(x+0.5)(x-7)}$

$y=x^2-7x+0.5x-3.5$

$\boxed{y=x^2-6.5x-3.5}$

Chapters 8 Financial Mathematics – Investing Money

1. Use the rule of 72.

$$t = \frac{72}{1.7} \doteq 42.4 \text{ years}$$

2.

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$120000 = P\left(1+\frac{0.0388}{2}\right)^{(2)(15)}$$

$$P = \frac{120000}{\left(1+\frac{0.0388}{2}\right)^{(2)(15)}} = \$67428.32$$

$$I = \$120000 - \$67428.32 = \$52571.68$$

3.

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$12000 = P\left(1+\frac{0.064}{12}\right)^{(12)(8)}$$

$$P = \frac{12000}{\left(1+\frac{0.064}{12}\right)^{(12)(8)}} = \$7201.34$$

4. $FV = \frac{Rn}{r}\left[\left(1+\frac{r}{n}\right)^{nt} - 1\right] = \frac{(800)(12)}{0.0203}\left[\left(1+\frac{0.0203}{12}\right)^{(12)(10)} - 1\right] = \$106338.86$