Jan. 5, 2016 Dec. 9, 2013 Nov. 27, 2012
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INVESTIGATE the Math

In Lesson 3.1, you discovered a side–angle relationship in acute triangles. Before this relationship can be used to solve problems, it must be proven to work in all acute triangles. Consider Ben's **proof**:

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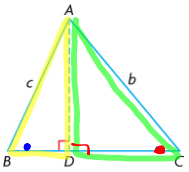
Pull for Lesson Notes

Using acute triangle ABC to prove the Sine Law



- Construct an acute triangle ABC

Step 1
I drew an acute triangle with height *AD*.

In $\triangle ABD$,

$$\sin B = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin B = \frac{AD}{c}$$

$$c \sin B = AD$$

In $\triangle ACD$,

$$\sin C = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin C = \frac{AD}{b}$$

$$b \sin C = AD$$

Step 2
I wrote equations for the sine of $\angle B$ and the sine of $\angle C$ in the two right triangles.

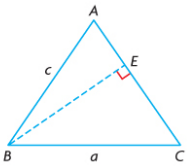
$$c \sin B = b \sin C$$

$$\frac{c \sin B}{\sin C} = b$$

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

Step 3
I set the expressions for *AD* equal to each other.

- If we construct an acute triangle ABC again, and used the line BE, we notice:



Step 4
I had expressions that involved sides *b* and *c* and $\angle B$ and $\angle C$, but I also needed an expression that involved *a* and $\angle A$. I drew a height from *B* to *AC* and developed two expressions for *BE*.

In $\triangle ABE$,

$$\sin A = \frac{BE}{c}$$

$$c \sin A = BE$$

In $\triangle CBE$,

$$\sin C = \frac{BE}{a}$$

$$a \sin C = BE$$

$$c \sin A = a \sin C$$

$$c = \frac{a \sin C}{\sin A}$$

Step 5
I set the expressions for *BE* equal to each other.

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Step 6
I set all three ratios equal to each other.

? How can you improve Ben's explanation of his proof?

A. Work with a partner to explain why Ben drew height *AD* in step 1.

B. In step 2, he created two different expressions that involved *AD*. Explain why.

C. Explain why he was able to set the expressions for *AD* equal in step 3.

D. Explain what Ben did to rewrite the equation in step 3.

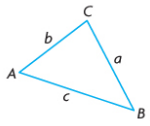
E. In steps 4 and 5, Ben drew a different height *BE* and repeated steps 2 and 3 for the right triangles this created. Explain why.

F. Explain why he was able to equate all three ratios in step 6 to create the **sine law**.

sine law

In any acute triangle,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



Answers

A.

B.

C.

D.

E.

F.

Law of Sines

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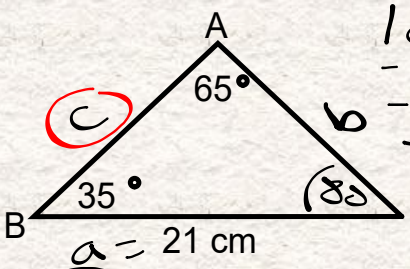
Used with an **oblique triangle** to find a missing side or angle.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Only **2** parts of this proportion will be used to solve each problem.

Example 1: Solve for **'c'**



$180 - 65 - 35 = 80$

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

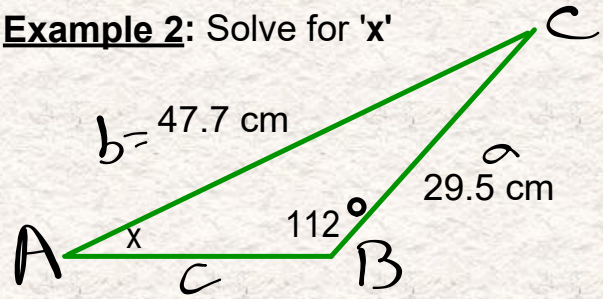
$$\frac{c}{\sin 80^\circ} = \frac{21}{\sin 65^\circ}$$

$$\frac{c \cdot \sin 65^\circ}{\sin 65^\circ} = \frac{21 \cdot \sin 80^\circ}{\sin 65^\circ}$$

$$c = \frac{21 \cdot \sin 80^\circ}{\sin 65^\circ}$$

$$c = 22.8 \text{ cm}$$

Example 2: Solve for **'x'**



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin A}{29.5} = \frac{\sin 112^\circ}{47.7}$$

$$\frac{47.7 \cdot \sin A}{47.7} = \frac{29.5 \cdot \sin 112^\circ}{47.7}$$

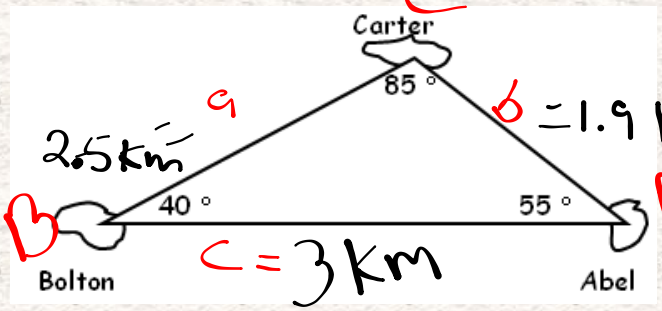
$$\sin A = \frac{29.5 \cdot \sin 112^\circ}{47.7}$$

$\angle A = \sin^{-1}(\text{ANS})$

$\angle A = 35^\circ$

Applications of Trigonometry

1. Abel Island and Bolton Island are 3 km apart. How far is Carter Island from Abel Island and Bolton Island on the given map?



C
a
b = 1.9 km
B
A
c = 3 km

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{a}{\sin 55^\circ} = \frac{3}{\sin 85^\circ}$

$a \cdot \sin 85^\circ = 3 \cdot \sin 55^\circ$

$\frac{a \cdot \sin 85^\circ}{\sin 85^\circ} = \frac{3 \cdot \sin 55^\circ}{\sin 85^\circ}$

$a = 2.5\text{km}$

$\frac{b}{\sin B} = \frac{c}{\sin C}$

$b = 1.9\text{ km}$

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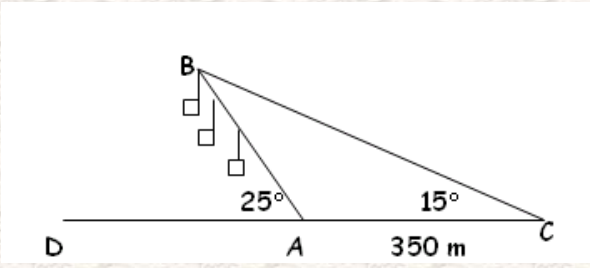

Attachments

PM11-3s2.gsp