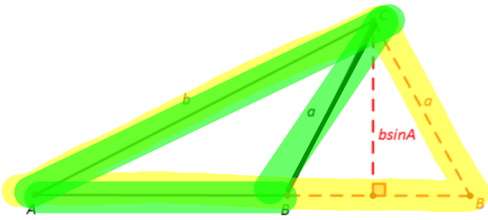
THE AMBIGUOUS CASE OF THE SINE LAW

Dec. 13, 2013 Dec. 4, 2012
Jan. 12, 2016
Dec. 14, 2017

The ***ambiguous case of the sine law*** is a situation in which ***two triangles can be drawn*** given the available information. The ambiguous case may occur when the given measurements are the lengths of *two sides and a non-contained angle* (SSA). In the diagram shown, two possibilities exist if we are given measurements for acute angle A, side a, and side b in a triangle. One possibility is that ∠B is *acute* and the other possibility is that ∠B is *obtuse.*



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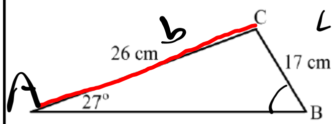
Example 1:

If we are to construct ΔABC where ∠A = 27°, a = 17 cm and b = 26 cm, the following cases are possible:

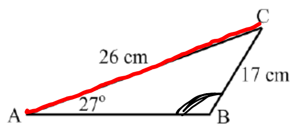
$$\frac{\sin B}{26} = \frac{\sin 27}{17}$$

$\angle B = 44$

Case 1:



Case 2:



It is important to note that for one sine value, there is more than one possible angle. *Supplementary* angles will have the same sine value, that is, sin θ = sin (180° − θ). If we know the sine of an angle and we use the calculator to determine the measure of that angle, we will get the *acute* angle possibility. If we know that the angle is *obtuse*, then we can calculate its measure by finding the *supplement* of the acute angle.

In the diagram above, using the sine law to solve for ∠B, we get:

So in **Case 1**, ∠B = 44° and in **Case 2**, ∠B = $180 - 44$ = 136°.

$\sin 44^\circ = 0.6946\ldots$

$\sin 136^\circ = 0.6946\ldots$

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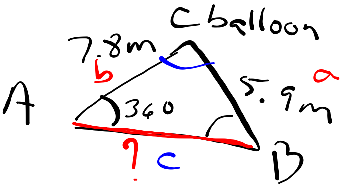
Example 2:

Albert and Belle are part of a scientific team studying thunderclouds. The team is about to launch a weather balloon into an active part of the cloud. Albert's rope is 7.8 m long and makes an angle of 36° with the ground. Belle's rope is 5.9 m long.

a) Is it necessary to consider the ambiguous case? Explain. Yes
b) Sketch the possible diagrams for this situation.
c) Determine the possible the distances between Albert and Belle to the nearest tenth of a metre.



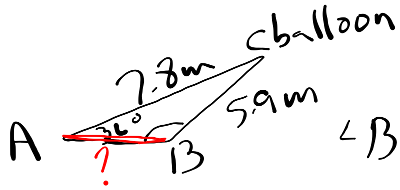
$\frac{\sin B}{7.8} = \frac{\sin 36°}{5.9}$

$\angle B = 51°$

$\angle C = 180 - 51 - 36 = 93°$

$\frac{c}{\sin 93} = \frac{5.9}{\sin 36°}$

$c = 10.0m$



$\angle B = 180 - 51$

$\boxed{\angle B\ 129}$

$\angle C = 180 - 129 - 36 = 15°$

$\frac{c}{\sin 15°} = \frac{5.9}{\sin 36°}$

$\boxed{c = 2.6m}$

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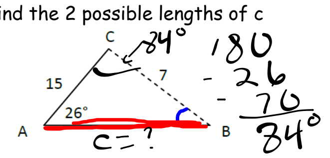
1. For ΔABC, $a = 7, b = 15$, and $m\angle A = 26°$. Find all possible $m\angle B$ to the nearest degree.

Find the 2 possible lengths of c



$180 - 26 - 70 = 84°$

$\frac{\sin \angle B}{b} = \frac{\sin \angle A}{a}$

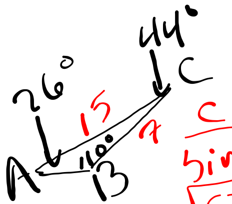
$\frac{\sin \angle B}{15} = \frac{\sin 26°}{7}$

$180 - 70 = 110°$

$\angle B = 70°, 110°$

$\frac{c}{\sin 84} = \frac{7}{\sin 26°}$

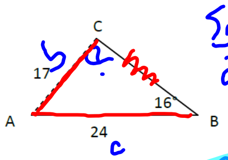
$\boxed{c = 15.9}$



$\frac{c}{\sin 44} = \frac{7}{\sin 26}$

$\boxed{c = 11.1}$

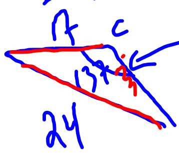
2. For ΔABC, $b = 17, c = 24$, and $m\angle B = 16°$. Find all possible $m\angle C$ to the nearest degree.



$\frac{\sin \angle C}{24} = \frac{\sin 16°}{17}$

$\angle C = 23°$

$180 - 23° = 157°$



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3. For ΔABC,
$a = 62, b = 53$, and $m\angle A = 54°$. Find all possible $m\angle B$ to the nearest degree.

3. $m\angle B = 44°$ & $136°$

4. A triangle has two sides with lengths of 20 and 15. The measure of the angle opposite the side with a length of 15 is 35°. Find all the possible measures of the angle opposite the side with a length of 20 to the nearest degree.

$\angle B = 50°$
$\angle B = 130°$

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Attachments

PM11-3s2.gsp