Module 1: Logical Reasoning

Reasoning Skills: Essential for Understanding Math

Nov. 18, 2013 Nov. 20, 2014

Nov. 23, 2015

Whether you realize it or not, you regularly use **logical reasoning** as you solve problems and reach certain conclusions. In fact, logical reasoning is an important method of thinking that enables us to come to useful conclusions with limited information. For example, if you walk into your classroom and see the desks in straight rows, each with a sharp pencil and a face-down booklet, your logical reasoning might lead you to conclude that your teacher has planned a surprise standardized test.

Logical reasoning is also essential in mathematics. Two basic methods of logical, mathematical reasoning are:

- inductive reasoning (bottom-up logic)
- deductive reasoning (top-down logic)

Both methods are useful for arriving at conclusions, and both are very important to the study of geometry.

Oct 27-2:30 PM

1.1 Making Conjectures: Inductive Reasoning

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GOAL Page 6

Use reasoning to make predictions.

Pull for Lesson Notes

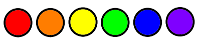
EXPLORE...

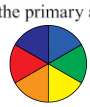
- If the first three colours in a sequence are red, orange, and yellow, what colours might be found in the rest of the sequence? Explain.

SAMPLE ANSWER

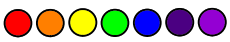
Here are three possible answers:

- If the colour sequence is red, orange, and yellow, the rest of the sequence may be green, blue, and purple. These colours are the primary and secondary colours seen on a colour wheel.

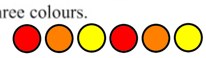




- If the colour sequence is red, orange, and yellow, the rest of the sequence may be green, blue, indigo, and violet. These colours are those of a rainbow.



- If the colour sequence is red, orange, and yellow, the rest of the sequence may repeat these three colours.



These ideas (or logical explanation) are what we call conjectures!

Conjecture *– based on evidence that you have gathered; the more support you have, the stronger your conjecture, but does not necessarily prove it*

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INVESTIGATE the Math Page 6 - A-D

Georgia, a fabric artist, has been patterning with equilateral triangles. Consider Georgia's **conjecture** about the following pattern.

conjecture
A testable expression that is based on available evidence but is not yet proved.

1

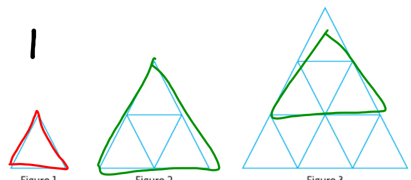

Figure 1 Figure 2 Figure 3

I think Figure 10 in this pattern will have 100 triangles, and all these triangles will be congruent to the triangle in Figure 1.

❓ How did Georgia arrive at this conjecture?

A. Organize the information about the pattern in a table.

Figure	1	2	3	4	5					
Number of Triangles	1	4	9	16	25					

Fill in 3, 4 & 5. Do you see a pattern?

B. With a partner, discuss what you notice about the data in the table.

C. Extend the pattern for two more figures.

D. What numeric pattern do you see in the table?

Answers

A.

Figure	1	2	3	4	5	6	7	8	9	10
Number of Triangles	1	4	9	16	25	36	49	64	81	100

B. The pattern in the table shows that the number of triangles equals the square of the figure number.

C.

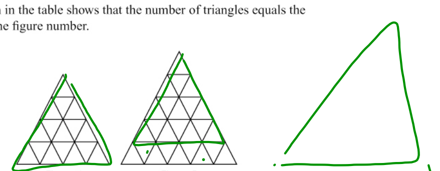

Figure 4 Figure 5

11 more

D. Figure 11 has 11^2 or 121 triangles.
Figure 12 has 12^2 or 144 triangles.

The numeric pattern in the table shows that each figure will have a perfect square of congruent triangles. The number of congruent triangles in each figure is the square of the figure number.

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Reflecting

E. Is Georgia's conjecture reasonable? Explain.

F. How did Georgia use **inductive reasoning** to develop her conjecture?

G. Is there a different conjecture you could make based upon the pattern you see? Explain.

* **inductive reasoning** *
Drawing a general conclusion by observing patterns and identifying properties in specific examples.

Answers

E. Georgia's conjecture is reasonable because, when the table is extended to the 10th figure, the pattern of values is the same as Georgia's prediction.

F. Georgia used inductive reasoning by gathering evidence about more cases. This evidence established a pattern. Based on this pattern, Georgia made a prediction about what the values would be for a figure not shown in the evidence.

G. A different conjecture could be made because a different pattern could have been seen. If the focus had been only on the congruent triangles with their vertices at the bottom and their horizontal sides at the top, then the following conjecture could have been made: The 5th figure will have 10 congruent triangles.

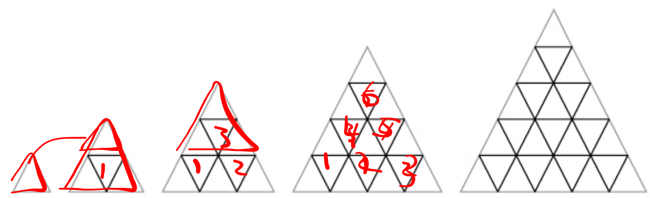

Figure 1 Figure 2 Figure 3 Figure 4 Figure 5

3 5 7 9

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EXAMPLE 2 Using inductive reasoning to develop a conjecture about integers

Make a conjecture about the product of two odd integers.

Jay's Solution

$(+3)(+7) = (+21)$	Odd integers can be negative or positive. I tried two positive odd integers first. The product was positive and odd.
$(-5)(-3) = (+15)$	Next, I tried two negative odd integers. The product was again positive and odd.
$(+3)(-3) = (-9)$	Then I tried the other possible combination: one positive odd integer and one negative odd integer. This product was negative and odd.
My conjecture is that the product of two odd integers is an odd integer.	I noticed that each pair of integers I tried resulted in an odd product.
$(-211)(-17) = (+3587)$	I tried other integers to test my conjecture. The product was again odd.

2, 4, 6, 8, 10 (+1 +1 +1 +1)

$2x \Rightarrow$ even #

$2x+1$

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EXAMPLE 3 Using inductive reasoning to develop a conjecture about perfect squares

Make a conjecture about the difference between consecutive perfect squares.

Ex1: $1^2 ; 2^2$ $3^2 ; 2^2$ $7^2 ; 8^2$

$4-1=3$ $9-4=5$ $64-49$

$2+1=3$ $=15$

$16-9=7$

$4+3=7$

$36-25=11$

$6+5=11$

Conjecture:

The difference between consecutive perfect squares results in an odd #.

12^2-11^2

$144-121=23$

$13^2-12^2 = 169 - 144 = 25$

$9^2 ; 10^2$

$100-81=19$

Nov 15-7:36 AM

EXAMPLE 3 Using inductive reasoning to develop a conjecture about perfect squares

Make a conjecture about the difference between consecutive perfect squares.

Francesca's Solution: Describing the difference numerically

$2^2 - 1^2 = 4 - 1$
$2^2 - 1^2 = 3$

I started with the smallest possible perfect square and the next greater perfect square: 1^2 and 2^2. The difference was 3.

$4^2 - 3^2 = 7$
$9^2 - 8^2 = 17$

Then I used the perfect squares 3^2 and 4^2. The difference was 7. So, I decided to try even greater squares.

My conjecture is that the difference between consecutive perfect squares is always a prime number.

I thought about what all three differences—3, 7, and 17—had in common. They were all prime numbers.

$12^2 - 11^2 = 23$

To test my conjecture, I tried the perfect squares 11^2 and 12^2. The difference was a prime number.

The example supports my conjecture.

Do you agree with Francesaca's conjecture?

$8^2 - 7^2$
$= 15$
Not prime

$19^2, 20^2$
$20^2 - 19^2 = 400 - 361$
$20 + 19 = 39$

In Summary

Key Idea

- Inductive reasoning involves looking at specific examples. By observing patterns and identifying properties in these examples, you may be able to make a general conclusion, which you can state as a conjecture.

Need to Know

- A conjecture is based on evidence you have gathered.
- More support for a conjecture strengthens the conjecture, but does not prove it.

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