7.1 Exploring Quadratic Relations

Pull for Lesson Notes

Oct. 14, 2014
Oct. 8, 2013
Oct. 16, 2015

GOAL

Determine the characteristics of quadratic relations.

Oct. 10, 2017

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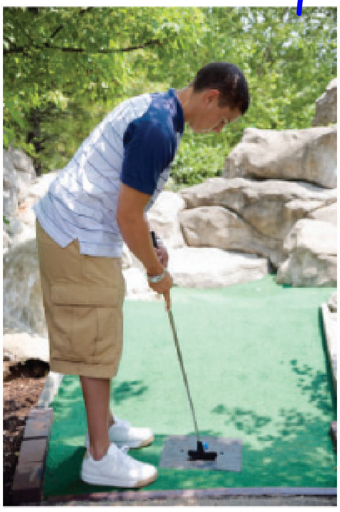
EXPLORE the Math



height y

x time

A moving object that is influenced by the force of gravity can often be modelled by a **quadratic relation** (assuming that there is no friction). For example, on one hole of a mini-golf course, the ball rolls up an incline after it is hit, slowing all the way due to gravity. If the ball misses the hole, it rolls back down the incline, accelerating all the way. If the initial speed of the ball is 6 m/s, the distance of the ball from its starting point in metres, y, can be modelled by the quadratic relation

$$y = -2.5x^2 + 6x$$

where x is the time in seconds after the ball leaves the putter.

quadratic relation
A relation that can be written in the standard form $y = ax^2 + bx + c$, where $a \neq 0$; for example, $y = 4x^2 + 2x + 1$

Graphing Technology

? How does changing the coefficients and constant in a relation that is written in the form $y = ax^2 + bx + c$ affect the graph of the relation?

of standard form of a quadratic

Ax+By+C=0

https://www.desmos.com/calculator

❓ How does changing the coefficients and constant in a relation that is written in the form $y = ax^2 + bx + c$ affect the graph of the relation?

Graphing Technology

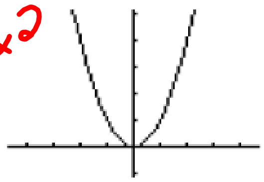
Sample Solution: Part 1

$y = 1x^2$

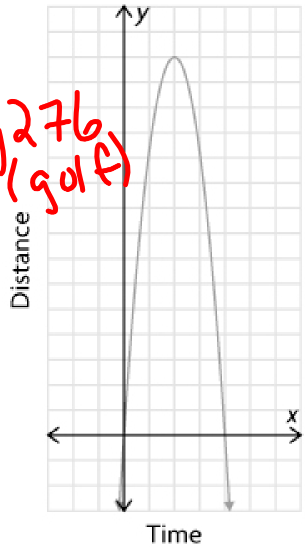
I know that the ball is rolled up an incline. The graph of this relation will go through the origin, because the ball will not move until it is struck by the golf club. The ball will continue to roll upward, slowing down because of gravity. Eventually, the ball will reverse direction and travel back down the incline if the person should miss the hole. My graph will represent the distance up the incline the ball reaches during the time it is on the incline.

I started with the basic function $y = 1x^2$. This corresponded to $a = 1$, $b = 0$, $c = 0$.

$y = x^2$

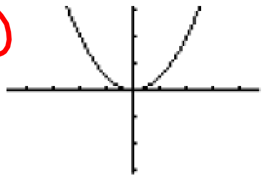


Pg 276 (golf)

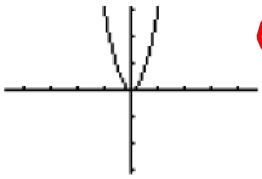


Technology:

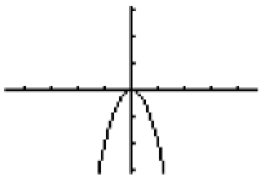
I then tried varying a while keeping b and c equal to 0. I noticed that the graph changed shape.

① 

$y = 1x^2$

② 

$y = 2x^2$

③ 

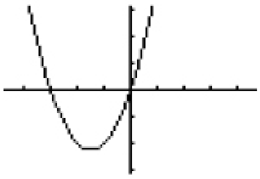
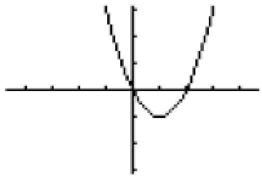
$y = -2x^2$

? How does changing the coefficients and constant in a relation that is written in the form $y = ax^2 + bx + c$ affect the graph of the relation?

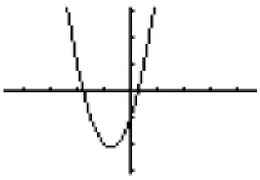
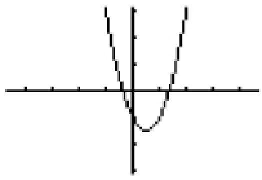
Graphing Technology

Sample Solution: Part 2

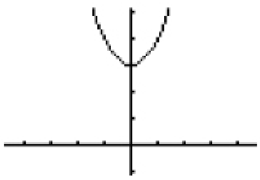
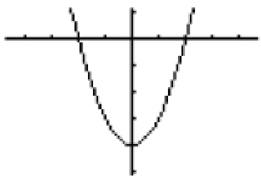
Next, I set $a = 1$, kept $c = 0$, and varied b. I noticed that the graph moved to the left or right, and up or down, but it did not change shape.

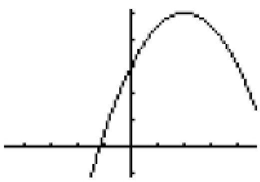
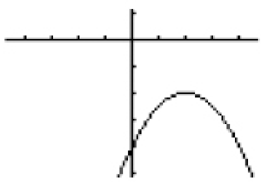
I also tried varying b but keeping $a = 2$ and $c = -1$ to see if it made a difference. I noticed the same effect on the graph.

Finally, I set $a = 1$ and $b = 0$ and varied c. I noticed that the graph moved up or down, but it did not change shape.

I also tried varying c but keeping $a = -0.5$ and $b = 2$ to see if it made a difference. I noticed the same effect on the graph.

Sept. 26, 2012

Reflecting

Pg 276

A. Describe the common characteristics of each of the **parabolas** you graphed.

B. Describe any symmetry in your graphs.

C. Are the quadratic relations that you graphed functions? Justify your decision.

D. What effects do the following changes have on a graph of a quadratic relation?

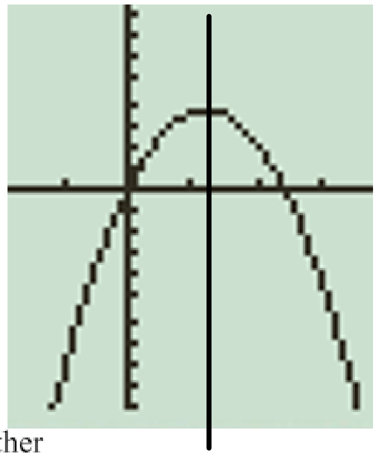
i) The value of a is changed, but b and c are left constant.

ii) ~~The value of b is changed, but a and c are left constant.~~

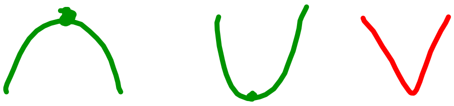
iii) The value of c is changed, but a and b are left constant.

parabola

The shape of the graph of any quadratic relation.



Answers



A. The parabolas all had a maximum or minimum point, they opened either up or down (never sideways), and they were all symmetrical about an axis that passes through the maximum or minimum point (vertex).

B. Looking at all of the graphs, the turning point has a connection to the symmetry of quadratic functions. If I draw a vertical line through this turning point as a line of reflection, I can see that the relation is a mirror image of itself on either side of this line.

C. Yes. None of the changes I made to a, b, and c result in a graph that fails the vertical-line test. I cannot locate any x-value that corresponds to two different y-values in the graphs I made.

D. **i)** The graph changes shape. Smaller/larger positive values of a make the graph wider/more narrow. Negative values of a turn the graph upside down.

ii) The graph shifts horizontally and vertically but does not change shape. As b becomes more positive/negative, the graph moves to the right/left and up/down.

iii) The graph shifts vertically but does not change shape. As c becomes more positive/negative, the graph moves up/down. I noticed that the value of c is always equal to the y-intercept of the graph.

Practice:

1. Rewrite these relations in standard form. Then, state whether each is quadratic (circle yes or no).

a) $y = 4 - x^2 + 2x$

$y =$ ____

Quadratic: yes/no -5

b) $y = 2(5x - 3)$

$y =$ ____

yes/no -5

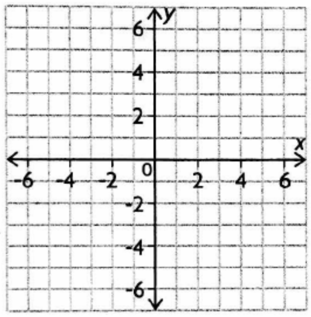
c) $y = (2x - 1)(x + 3)$

$y =$ ____

yes/no -5

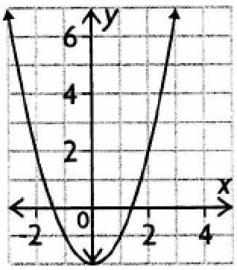
2. For the relations in question 1 that are quadratic, state the direction of opening of the parabola and its y-intercept.

3. A quadratic function is defined by $y = 2x^2 - 4x + c$. As the value of c varies, predict what happens to the line of symmetry and the lowest point on the parabola. Check your predictions by graphing several functions on the grid provided.



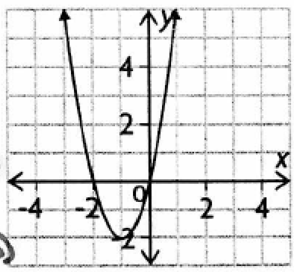
4. Identify the value of c for each parabola.

a)



$c =$ ____

b)



Xint -2 and 0

$(x+2)(x+0)$

x^2+2x

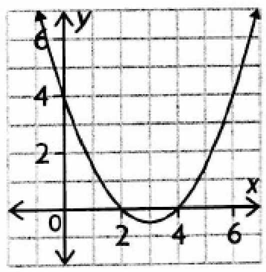
$a=1$

$b=2$

$c=0$

$c =$ ____

c)



$c =$ ____

5. This table of values lists points in a quadratic relation.

a) What is the y-intercept of the parabola?

Is this the highest or lowest point on the parabola?

b) Without graphing, predict the direction in which the parabola opens. Explain how you know.

x	y
−3	1
−2	−2
−1	−3
0	−2
1	1
2	6

Practice

1. Rewrite these relations in standard form. Then, state whether each is quadratic (circle yes or no).

a) $y = 4 - x^2 + 2x$
$y = -x^2 + 2x + 4$
Quadratic: yes/no

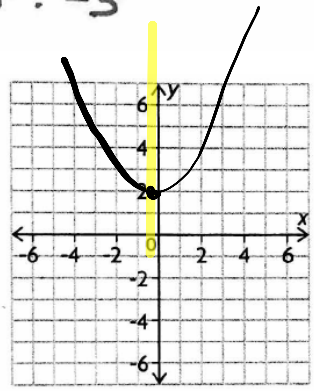
b) $y = 2(5x - 3)$
$y = 10x - 6$
yes/no

c) $y = (2x - 1)(x + 3)$
$y = 2x^2 + 5x - 3$
yes/no

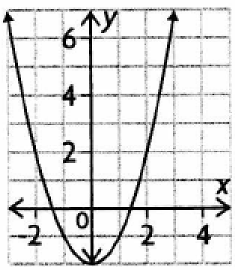
2. For the relations in question 1 that are quadratic, state the direction of opening of the parabola and its *y*-intercept.

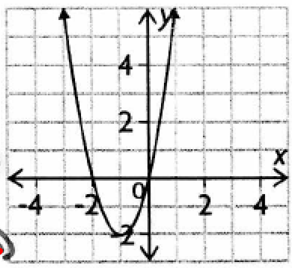
a) opens downwards, y-int = +4
c) opens upwards, y-int: −3

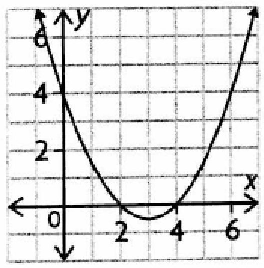
3. A quadratic function is defined by $y = 2x^2 - 4x + c$. As the value of *c* varies, predict what happens to the line of symmetry and the lowest point on the parabola. Check your predictions by graphing several functions on the grid provided.



4. Identify the value of *c* for each parabola.

a)

c = −2

b)

c = 0

c)

c = 4

5. This table of values lists points in a quadratic relation.

a) What is the *y*-intercept of the parabola? −2

Is this the highest or lowest point on the parabola? neither

b) Without graphing, predict the direction in which the parabola opens. Explain how you know.

upwards, lowest point is −3 (y-value)

x	*y*
−3	1
−2	−2
−1	−3
0	−2
1	1
2	6

Don't copy
(Page 277)

Video

In Summary

Key Ideas

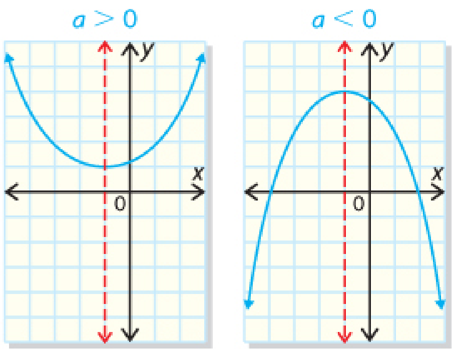
- The degree of all quadratic functions is 2.
- The standard form of a quadratic function is

$$y = ax^2 + bx + c$$

  where $a \neq 0$.
- The graph of any quadratic function is a parabola with a single vertical line of symmetry.

Need to Know

- A quadratic function that is written in standard form, $y = ax^2 + bx + c$, has the following characteristics:
  - The highest or lowest point on the graph of the quadratic function lies on its vertical line of symmetry.
  - If a is positive, the parabola opens up. If a is negative, the parabola opens down.



  - Changing the value of b changes the location of the parabola's line of symmetry.
  - The constant term, c, is the value of the parabola's y-intercept.

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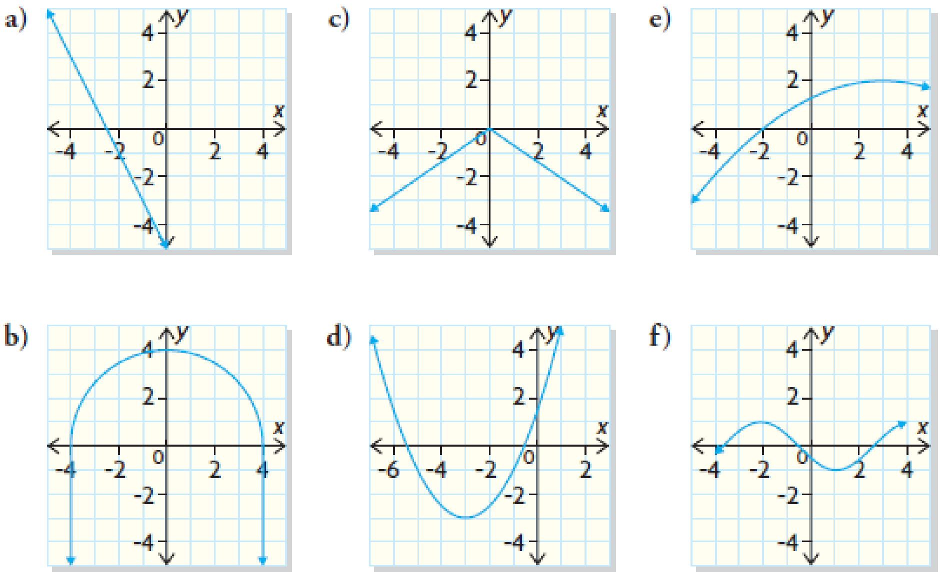
FURTHER Your Understanding

1. Which graphs appear to represent quadratic relations? Explain.



Attachments

FM11-7s1.gsp