Compound Interest

Compound Interest: The interest that is earned or paid on both the principal *and* the accumulated interest.

Compounding Period: The time over which interest is determined.

Interest can be compounded annually, semi-annually (every 6 months), quarterly (every 3 months), monthly, weekly, or daily.

Compound Interest Formula: $A = P\left(1 + \frac{r}{n}\right)^{nt}$ (FV)

A = future value
P = principal or present value
r = interest rate per year
n = number of compounding periods per year
t = time in years

Rule of 72: A formula for *estimating* the time it will take for an investment to double. The Rule of 72 is most accurate when the interest is compounded annually.

$$\text{Years to Double} = \frac{72}{\text{Interest Rate}}$$

Total Interest Earned: $I = A - P$

Rate of Return Formula: $\text{Rate of return} = \frac{I}{P} = \frac{A - P}{P}$

Example 1: Determining the Future Value of an Investment

Matt has invested a $23 000 inheritance in an account that earns 13.6%, compounded semi-annually. The interest rate is fixed for 10 years. Matt plans to use the money for a down payment on a house in 10 years.

a) What is the future value of the investment after 10 years?

b) What total interest will Matt have earned?

Solution:

a) $FV = P(1+\frac{r}{n})^{nt}$

$= 23\,000(1+\frac{0.136}{2})^{2 \cdot 10}$

$= 23\,000(1.068)^{20}$

$= \$85\,733.96$

DO NOT ROUND !!!!!

b) $I = \underset{FV}{A} - P$

$= \$85\,733.96 - 23\,000$

$= \$62\,733.96$

Example 2: Estimating Doubling Times for Investments

Kris invested $5000 by purchasing a Canada Savings Bond which earns 9%, compounded annually. Estimate the doubling time for his CSB.

Solution:

$$\text{years to double} = \frac{72}{9}$$

$$= 8 \text{ yrs}$$

Check: $FV = P\left(1+\frac{r}{n}\right)^{nt}$

$$= 5000\left(1+\frac{0.09}{1}\right)^{1 \cdot 8}$$

$$= 5000(1.09)^8$$

$$= \$9962.81$$

**** Practice: Pages 468-469 Questions 2, 5, 6, 8, 11***

2. Determine the future value and the total interest earned for each investment.
- **a)** $520 invested for 8 years at 4.5% compounded monthly
- **b)** $1400 invested for 15 years at 8.6% compounded semi-annually

5. Parker wanted to buy a new motorcycle but he had only $6000, half the amount he needed.
- **a)** Estimate when Parker could buy the motorcycle if he invested his money at 4.8%, compounded annually. Verify your estimate.
- **b)** Estimate how much sooner he could buy the motorcycle if his investment earned 7.2%, compounded annually. Verify your estimate.

6. Trust funds are investments that are set up for a specific purpose. A local business invested $250 000 in a charitable trust fund so that a school can offer scholarships. The interest rate is 3.8%, compounded semi-annually. Only the interest earned can be used to provide the scholarships. How much is available from the trust fund for scholarships each year?

8. Estimate how long it would take for $1000 to grow to $16 000 at each interest rate, compounded annually.
- **a)** 6%
- **b)** 12%

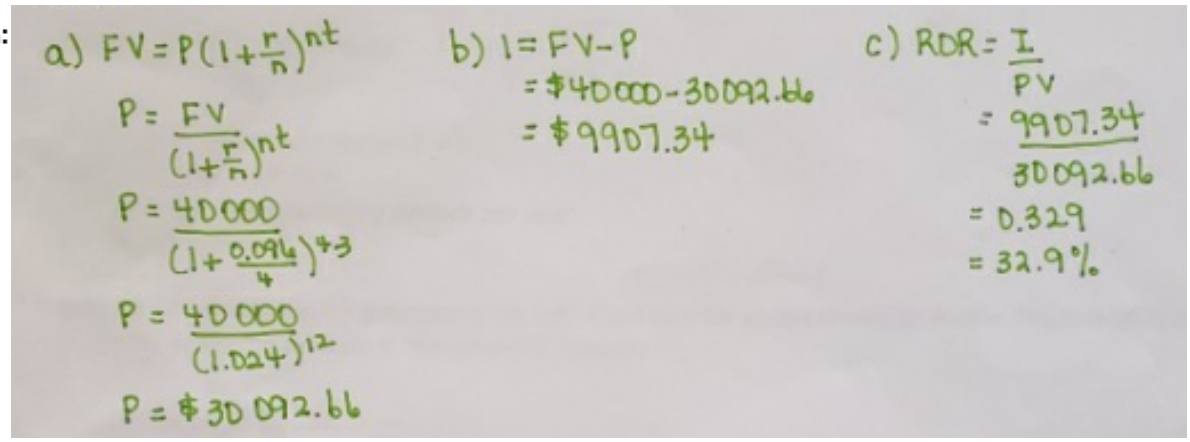
11. On Freda's 16th birthday, she invested $1500 in an account that earns 9%, compounded semi-annually. On her 20th birthday, she moved her investment to an account that paid 11%, compounded monthly. Determine the value of her account on her 22nd birthday.

Example 3: Determining the Present Value of an Investment

Agnes and Bill are musicians. They have researched the costs to set up a small recording studio. They estimate that $40 000 will pay for the soundproofing, recording equipment, and computer hardware and software that they need. They plan to set up the studio in 3 years and have invested money at 9.6%, compounded quarterly, to save for it.

a. How much money should they have invested?
b. How much interest will they earn over the term of their investment?
c. Calculate what their rate of return would be.

Solution:



* Practice: Pages 478-479 Questions 4, ~~5a~~, 6, 7, ~~11a~~

4. Mac plans to retire in 20 years, when he is 55. He estimates that he will need $250 000 to live on, until he is eligible for his pension.

a) How much money should he invest now, at 8.5% compounded annually, to meet his goal?

b) How much interest will he earn in 20 years?

a) $FV = P\left(1+\frac{r}{n}\right)^{nt}$

$P = \frac{FV}{\left(1+\frac{r}{n}\right)^{nt}}$

$= \frac{250\,000}{\left(1+\frac{0.085}{1}\right)^{1 \cdot 20}}$

$= \frac{250\,000}{5.112\ldots}$

$= \$48904.10$

b) $I = FV - P$

$= 250000 - 48904.10$

$= \$201\,095.90$

6. Claire wants a down payment of $17 500 to buy a house in 10 years, when she turns 30. Her bank offers her an investment with 5.6% interest, compounded semi-annually. What present value will she need to invest now?

7. Sasha predicts that she will need $24 000 to remodel her carpentry workshop in 6 years. She has found three investment options to consider:

A. 4.80%, compounded annually
B. 4.75%, compounded semi-annually
C. 4.70%, compounded quarterly

a) Compare the rates of return for these three options. Which option should she choose? Why?
b) How much interest will she earn?